Whispering Gallery Modes and Plasmons in optical cavities

Monique Dauge
University of Rennes 1 and CNRS, Rennes, FRANCE
monique.dauge@univ-rennes1.fr

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Resonators in optics are mainly build from compact objects called cavities. At microscale (a few wave lengths) theses cavities are made from a dielectric material such as glass, whereas at nanoscale, cavities are made of metal such as gold. The principal physical characteristics of the cavity is its electric permittivity ε_{c}, which is real and > 1 for dielectric materials, and negative for metals (in suitable frequency range).

A step in the study of such devices is the investigation of complex scattering frequencues $k \in \mathbb{C}$ of the open system formed by a bidimensional cavity Ω embedded in the free space, in our case the Helmholtz equation in $\mathbb{R}^2$:

$$(1) \qquad \begin{cases} -\operatorname{div} \dfrac{1}{\varepsilon} \nabla u = k^2 u & \text{in} \quad \mathbb{R}^2, \\ \text{outgoing radiation condition} & |x| \to \infty. \end{cases}$$

Here ε has a jump across $\partial\Omega$:

$$\varepsilon = \begin{cases} \varepsilon_{\mathsf{c}} & \text{in} \quad \Omega, \\ 1 & \text{in} \quad \mathbb{R}^2 \setminus \overline{\Omega}. \end{cases}$$

so system (1) is a transmission problem.

In optical devices, the modes of interest are those which concentrate along the boundary of Ω. For dielectric cavities, these are the whispering gallery modes. For negative index cavities, plasmons also have this property.

In this talk, I mainly address whispering gallery modes (WGM) with focus on radially symmetric cavities with radially varying permittivity. We show how asymptotic analysis and numerical computations describe WGM as the angular frequency of modes m tends to infinity [1, 2].

Relying on the recent work [3], I will also address plamonic modes in the case of a negative index material and show similarites and contrasts with whispering gallery modes.

References

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[2] S. Balac, M. Dauge, and Z. Moitier, *Asymptotics for 2D whispering gallery modes in optical micro-disks with radially varying index*, IMA Journal of Applied Mathematics, **86** (2021), no. 6, 1212-1265. arXiv: `https://arxiv.org/abs/2003.14315`

[3] C. Carvalho and Z. Moitier, *Scattering resonances in unbounded transmission problems with sign-changing coefficient* (2022), arXiv: `https://arxiv.org/abs/2010.07583`