

Sharp weighted first and second order Caffarelli-Kohn-Nirenberg type inequalities in the L^2 setting

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In this exposure we first present uncertainty principles with radially symmetric weights applied to curl-free vector fields. Part of these new inequalities extend the family of Caffarelli-Kohn-Nirenberg (CKN) inequalities studied by Catrina and Costa (J. Differential Equations 2009) from scalar fields to curl-free vector fields. The best constants are improved. We will apply a new representation of curl-free vector fields developed by Hamamoto (J. Funct. Anal. 2021). The newly obtained results are also sharp and minimizers are described explicitly. Secondly, we prove new sharp second order interpolation functional inequalities for scalar fields with radial weights. We apply new factorization methods by adding some new crucial terms in our square expansion formula. In some situations we combine this method with a method based on decomposition in spherical harmonics.

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References

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