The Monotonicity of the p-Torsional Rigidity in Convex Domains

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For any bounded and convex set $\Omega \subset \mathbb{R}^N$ $(N \geq 2)$ with smooth boundary, $\partial\Omega$, and any real number $p > 1$, we denote by u_p the *p-torsion function* on Ω, that is the solution of the *torsional creep problem*

$$\begin{cases} -\Delta_p u = 1 & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases} \tag{1}$$

where $\Delta_p u := div(|\nabla u|^{p-2}\,\nabla u)$ is the p-Laplace operator. The aim is to investigate the monotonicity with respect to p for the *p-torsional rigidity* on Ω, defined as

$$T_p\,(\Omega) := \int_\Omega u_p dx.$$

More precisely, we show that there exist two constants $D_1 \in \left[\frac{1}{2}, e^{\frac{-1}{N+1}}\right]$ and $D_2 \in [1, N]$ such that for each bounded and convex set $\Omega \subset \mathbb{R}^N$ with $\frac{|\partial\Omega|}{|\Omega|} \leq D_1$, the function $p \to T_p(\Omega)$ is decreasing on $(1, \infty)$ while for each bounded and convex set $\Omega \subset \mathbb{R}^N$ with $\frac{|\partial\Omega|}{|\Omega|} \geq D_2$, the function $p \to T_p(\Omega)$ is increasing on $(1, \infty)$. Moreover, for each real number $s \in (D_1, D_2)$ there exists a bounded and convex set $\Omega \subset \mathbb{R}^N$ with $\frac{|\partial\Omega|}{|\Omega|} = s$ such that the function $p \to T_p(\Omega)$ is not monotone on $(1, \infty)$.

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