Conditional propagation of chaos for interacting particle systems in a diffusive regime

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We consider a stochastic system of interacting particles (neurons, economical agents) in a diffusive scaling. The system consists of N particles, each jumping randomly with state dependent rate. At the jump times, the state of the jumping particle looses memory of its past. At the same time, all other particles receive an additional jump height which is a centred random variable of order $1/\sqrt{N}$, added to their state. In between successive jumps, the particles evolve according to a deterministic flow. We prove the convergence of the system, as $N \to \infty$, to a limit nonlinear jumping stochastic differential equation driven by Poisson random measure and an additional Brownian motion W which is created by the central limit theorem. This Brownian motion is underlying each particle's motion and induces a common noise factor for all particles in the limit system. Conditionally on W, the different particles are independent. This is the *conditional propagation of chaos* property. We first discuss the well-posedness of the limit equation. To prove the convergence in distribution of the finite system to the limit system, we introduce a new martingale problem that is well suited for our framework. The uniqueness of the limit is deduced from the exchangeability of the underlying system

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