Unique continuation for convection–diffusion equations using stabilised finite elements

Mihai Nechita
Tiberiu Popoviciu Institute of Numerical Analysis
Romanian Academy, Cluj-Napoca, Romania
mihai.nechita@ictp.acad.ro

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We consider the unique continuation problem for a stationary convection–diffusion equation, with data given in an interior subset of the domain and no boundary conditions. For this ill-posed problem, we first discuss conditional stability estimates that are explicit in the physical parameters. Casting the problem as pde-constrained optimisation, we present a finite element method based on a discretise-then-regularise approach. The regularisation is based on penalising the jumps of the gradient across the interior faces of the finite element triangulation.

When diffusion dominates, we prove convergence rates by applying the continuum stability estimates to the approximation error and controlling the residual through stabilisation. When convection dominates, we perform a local analysis and obtain weighted error estimates with quasi-optimal convergence along the characteristics of the convective field through the data set.

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Références

[1] E. Burman, M. Nechita, L. Oksanen, ***A stabilized finite element method for inverse problems subject to the convection–diffusion equation. I: diffusion-dominated regime***, Numer. Math., 144:451–477, 2020.

[2] E. Burman, M. Nechita, L. Oksanen, ***A stabilized finite element method for inverse problems subject to the convection–diffusion equation. II: convection-dominated regime***, Numer. Math., 150:769–801, 2022.