Functional convex ordering of stochastic processes : from Brownian diffusions to stochastic Volterra equations

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After some short background on convex ordering of $\mathbb{R}^d$-valued random vectors, its connections with martingales through Kellerer's and Strassen's theorems, we show how to establish functional convex ordering for (scaled) Brownian diffusions sharing the same drift based on the pointwise ordering of their diffusion coefficient. Our approach is constructive : we first establish our results in discrete time for the Euler discretization scheme and then transfer them using some functional limit theorems à la Jacod-Shiryaev. This approach is a kind of paradigm and can be applied to many other processes (e.g. Lévy driven diffusions or of McKean-Vlasov type) and functionals (Snell envelope with applications in Finance, etc). A natural question at this stage is to wonder whether it is possible to apply an approach to (non-martingale and) non-Markovian dynamics like solutions to stochastic Volterra equations. A positive answer is provided in. ajoint work with B. Jourdain under convexity assumptions on the coefficients, however stronger then those needed for regular diffusions. These results admit extensions to monotone convex ordering. This last result has applications in finance to futures on VIX (volatility index) in (quadratic) rough volatility models.

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