Stochastic convex orders and applications

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Let $p_{n,j}(x) := \binom{n}{j} x^j (1-x)^{n-j}$, $x \in [0,1]$, $0 \leq j \leq n$. The analytic inequality

$$\sum_{i=0}^{n} \sum_{j=0}^{n} \left[p_{n,i}(x) p_{n,j}(x) + p_{n,i}(y) p_{n,j}(y) - 2 p_{n,i}(x) p_{n,j}(y) \right] f\left(\frac{i+j}{2n} \right) \geq 0,$$

valid for each convex function $f \in C[0,1]$, is the simplest illustration of the results presented in this talk. It is related with the shape preserving properties of the Bernstein-Schnabl operators, see [4, Sec. 3.4]. Its first proof [6] uses stochastic convex orderings. The first *analytic* proof [1] was followed by many other proofs, in analytic or probabilistic terms, involving more general families of operators and convex functions of higher order, see [2], [5] and the references therein. The talk surveys the existing results in this area and presents some new, very recent results and problems [3].

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Références

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