From variational Mean field Games to trajectory optimization in L^1

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I will first present the general ideas of mean-field games theory, as it was introduced some fifteen years ago by J.-M. Lasry and P.-L . Lions, focusing in particular on the case where agents seek to avoid congestion due to too high density, which is found in several models of road or pedestrian traffic. Under certain assumptions on the structure of the cost, the game is a potential game, where the equilibrium can be found by minimizing a global energy, typically convex. In particular, when the cost minimized by each agent contains the integral of its squared velocity, we obtain a variational problem in the Wasserstein space W_2 which is very reminiscent of the dynamic formulation *à la* Benamou-Brenier of the optimal transport problem. After this introduction, I will explain in particular why it is crucial to pay attention to the regularity of the solutions in order to rigorously associate an equilibrium with each solution of the variational problem, and in which sense. Then, motivated by the search for a reasonable MFG model for the real estate market, I will present a variant, where the cost minimized by the agents is quite different in nature : their trajectories are constrained to be piecewise constant and the cost includes the number of jumps (this represents the evolution of the agents in physical space, and each jump stands from moving from one address to another). The corresponding variational problem involves, instead of the Wasserstein distance, the total variation distance between measures or, when dealing with densities, the L^1 distance. I will then come back to the regularity question, which is very relevant in this setting for several reasons : not only for the equivalence between optimizers and equilibria, but also to prove that, despite the discontinuous behavior of the agent trajectories, their density evolves in a smooth way, in accordance with what can be observed in reality.

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Références

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