Metastability in the viscous Burgers equation, an entropy approach

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First observed by Kim and Tzavaras [1] roughly to decades ago, metastability in the viscous Burgers equation

$$u_t + uu_x = \varepsilon u_{xx}$$

refers to the peculiar dynamic phenomenon that for small $\varepsilon \ll 1$ the evolution consists of two stages : a) a fast initial transient during which an N-wave is formed (almost identical to the inviscid $\varepsilon = 0$ case) followed by b) a very slow decay of the N-profile to a diffusion wave, the system's asymptotic state.

Subsequently, Beck and Wayne [2] have given a rigorous treatment of the problem, which they regard as a toy model for Navier-Stokes. In this contribution, we propose a new approach to the analysis of metastabilty based on relative entropy in the sense of [3].

As pointed out in [4], the phenomenon of metastability is quite relevant to the numerical integration of PDEs, from the perspective of long-time behavior of numerical schemes. This aspect is also considered.

Références

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